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ON THE OSCILLATIONS OF A THIN-WALLED STRUCTURE CONTAINING A FLUID, IN THE PRESENCE OF A HYDRODYNAMIC DAMPER

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A model of a hydrodynamic oscillation damper is proposed. The model is used to obtain the equations describing longitudinal oscillations of a structure which includes a shell partially filled with fluid, and contains a hydrodynamic damper. It is shown that the use of the damper leads to considerable increase in the damping of the oscillations of specified frequencies within the structure.

In modern technology one encounters various types of problems connected with restricting the amplitudes of the axisymmetric vibrations of shells and of the longitudinal oscillations of structures consisting of shells partially filled with fluid. Various devices have been proposed [1] for solving these problems. All these devices have a common feature, namely an elastic shell filled with gas and placed in the fluid. The natural frequency of oscillations of such a shell in a fluid can be tuned to required frequency. The effect of such a device is analogous to the effect of a dynamic vibration damper in mechanical systems [2]. A part of the fluid contained in the shell serves as the active mass of the dynamic damper, and for this reason we shall call such devices the hydrodynamic vibration dampers.

1. Formulation of the problem. Let us consider a model of a hydrodynamic damper in which the shell consists of a bellows with a rigid lid at each end (Fig. 1). The lower lid is joined to the reference frame by means of sufficiently stiff rods, and the upper lid is movable. A damper with viscous friction is placed in the gas cavity thus formed.

We shall study the influence of the hydrodynamic damper on the axisymmetric oscillations of the shell partially filled with fluid, and on the simultaneous longitudinal oscillations of the construction housing and the axisymmetric, longitudinal-transverse oscillations of the shell with fluid. The latter problem will be solved for an axisymmetric elastic construction a part of which consists of a shell partially filled with fluid containing within it a hydrodynamic damper. We shall model our computations on the elastic rod with attached oscillators [3].

We introduce the following assumptions. The fluid is perfect and incompressible. Generation of waves on the free surface and the change in the hydrostatic pressure caused by the deformation of the shell can both be neglected. The unperturbed free surface of the fluid is perpendicular to the longitudinal Ox-axis, and the axis points in the direction opposite to that of the mass force field vector. The reference frame of the shell is perfectly rigid.

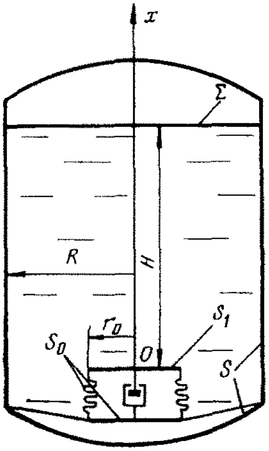

Fig. 1

2. Equations of oscillations of the shell with a hydrodynamic damper. We denote the displacement of the shell frame along the Ox-axis by $u\ (t)$, and the deviation of the upper lid of the bellows from its position of equilibrium by $\zeta\ (t)$. The equations of natural oscillations of the shell at $\zeta = 0$ have the form [3]

$$(m_0 + m)\, u^{\cdot\cdot} + \sum_{n=1}^{\infty} \lambda_{n0} u_n^{\cdot\cdot} = 0 \tag{2.1}$$

$$\mu_{n0}\, (u_n^{\cdot\cdot} + \omega_{n0}^2 u_n) + \lambda_{n0} u^{\cdot\cdot} = 0$$

$$(n = 1, \ldots, \infty)$$

where m_0 and m denote the mass of the shell and fluid, respectively, λ_{n0} and μ_{n0} are the apparent additional masses of fluid, ω_{n0} are the partial frequencies of the axisymmetric oscillations of the shell with a fluid and n is the number of the oscillation mode. We can adopt, as the generalized coordinates u_n, the fictitious displacements of the free surface of the fluid when the shell is in its n-th order oscillations, with the reference frame rigidly clamped.

Let us put a certain Lagrangian function $L_0 = T_0 - \Pi_0$ in one-to-one correspondence with the equations (2.1). We write the Lagrangian function of a shell with a hydrodynamic damper in the form $L = L_0 + L_1$ and find L_1. Let us introduce the fluid displacement potential

$$\chi = \chi_0 + \zeta V, \quad \chi_0 = ux + \sum_{n=1}^{\infty} u_n \Phi_n \tag{2.2}$$

The functions Φ_n and V are harmonic and satisfy the boundary conditions

$$\Phi_n = 0 \text{ on } \Sigma,\ \partial\Phi_n/\partial\nu = w_n \text{ on } S,\ \partial\Phi_n/\partial\nu = 0 \text{ on } S_0 + S_1$$
$$V = 0 \text{ on } \Sigma,\ \partial V/\partial\nu = 1 \text{ on } S_1, \quad \partial V/\partial\nu = 0 \text{ on } S + S_0$$

where S and Σ denote the surface of the shell and the free surface of the fluid, S_1 and S_0 denote the surface of the upper lid of the bellows and the fixed surface of the damper, ν is the unit vector of the outer normal to the surface enclosing the volume τ occupied by the fluid, and w_n are the partial forms of the oscillations of the shell partially filled with fluid.

Let us write the expressions for the kinetic and potential energy of the mechanical system in question

$$T = T_0 + \frac{1}{2}\rho \int_\tau \left(\nabla \frac{\partial\chi}{\partial t}\right)^2 d\tau - \frac{1}{2}\rho \int_\tau \left(\nabla \frac{\partial\chi_0}{\partial t}\right)^2 d\tau, \quad \Pi = \Pi_0 + \frac{1}{2} c\zeta^2$$

Here ρ and c denote the fluid density and rigidity of the damper. Taking into account (2.2), we find

$$L_1 = \frac{1}{2} b\zeta^{\cdot 2} + \varepsilon_0 u^{\cdot}\zeta^{\cdot} + \sum_{n=1}^{\infty} \varepsilon_{n0}\zeta^{\cdot} u_n^{\cdot} - \frac{1}{2} c\zeta^2 \tag{2.3}$$

$$b = \rho \int_\tau (\nabla V)^2 d\tau, \quad \varepsilon_0 = \rho \int_\tau \nabla x \nabla V d\tau, \quad \varepsilon_{n0} = \rho \int_\tau \nabla V \nabla \Phi_n d\tau$$

We note that [3]

$$\mu_{n0} = \rho \int_\tau (\nabla \Phi_n)^2 d\tau, \qquad \lambda_{n0} = \rho \int_\tau \nabla x \nabla \Phi_n d\tau$$

Taking into account (2.1), we can write the Lagrange's equations in the form

$$(m_0 + m) u^{\cdot\cdot} + \sum_{n=1}^{\infty} \lambda_{n0} u_n^{\cdot\cdot} + \varepsilon_0 \zeta^{\cdot\cdot} = 0 \tag{2.4}$$

$$\mu_{n0} (u_n^{\cdot\cdot} + \omega_{n0}^2 u_n) + \lambda_{n0} u^{\cdot\cdot} + \varepsilon_{n0} \zeta^{\cdot\cdot} = 0 \quad (n = 1, \ldots, \infty)$$

$$b(\zeta^{\cdot\cdot} + \omega_\zeta^2 \zeta) + \varepsilon_0 u^{\cdot\cdot} + \sum_{n=1}^{\infty} \varepsilon_{n0} u_n^{\cdot\cdot} = 0, \quad \omega_\zeta^2 = \frac{c}{b}$$

where the inherent mass of the damper is neglected.

When $\omega_\zeta \ll \omega_{10}$, putting in (2.4) $u_n = 0$ yields the equations of motion of a rigid shell with a hydrodynamic damper. The equations enable us to bring in a simple mechanical analog including the main mass $M = m_0 + m - m_1$ and the auxilliary mass $m_1 = \varepsilon_0^2/b$ connected to M by means of a spring with rigidity of $c_1 = c\varepsilon_0^2/b^2$. We shall show below that the efficiency of the damper depends on the magnitude of m_1.

Let us estimate m_1 for a cylindrical shell with a hollow bottom. We introduce a cylindrical $Oxr\theta$ -coordinate system with the origin at the center of the top of the bellows lid. We formulate the boundary value problem for the potential V in the following approximate form:

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial x^2} = 0 \text{ in } \tau \tag{2.5}$$

$$\partial V / \partial r = 0, \ r = R; \quad V = 0, \ x = H$$

$$\frac{\partial V}{\partial x} = \begin{cases} 1, \ x = 0, \ 0 \leqslant r \leqslant r_0 \\ 0, \ x = 0, \ r_0 < r \leqslant R \end{cases}$$

where H is the height of the column of fluid above the bellows, R is the radius of the shell and r_0 the radius of the bellows.

Using the Fourier method of separating the variables in (2.5), we obtain

$$V(x, r) = \frac{(x - H) r_0^2}{R^2} + 2R \sum_{n=1}^{\infty} \frac{r_0 J_1 (\xi_n r_0 / R)}{R \xi_n^2 J_0^2 (\xi_n)} \frac{\operatorname{sh} [\xi_n (x - H)/R]}{\operatorname{ch} (\xi_n H)/R)} J_0 \left(\frac{\xi_n z}{R}\right)$$

where J_0 and J_1 are Bessel functions of the first kind, of the zero and first order, and ξ_n are the roots of the equation $J_1(\xi) = 0$. Applying to (2.3) the Green's formula and taking into account the boundary conditions (2.5), we obtain

$$b = \pi\rho \frac{r_0^4 H}{R^2} + 4\pi\rho R^3 \sum_{n=1}^{\infty} \frac{\operatorname{th}(\xi_n H/R)}{\xi_n^2} \frac{r_0^2 J_1^2(\xi_n r_0/R)}{R^2 J_0^2(\xi_n)}$$

$$\varepsilon_0 = \pi\rho r_0^2 H$$

Fig. 2 shows the results of the computations. The curves *1* and *2* depict the dependence of the dimensionless associated masses $\overline{m}_1(r_0/R) = m_1/(\pi\rho R^3)$ and $\overline{b}(r_0/R) = b/(\pi\rho R^3)$ for $H/R = 2$ and $H/R = 3$, respectively. We see that the mass m_1 represents a considerable proportion of the total mass of fluid in the shell in the case when the hydrodynamic damper is relatively small, the latter fact governed by the smallness of the associated mass b. This fact is of considerable importance in the application of the hydrodynamic damper.

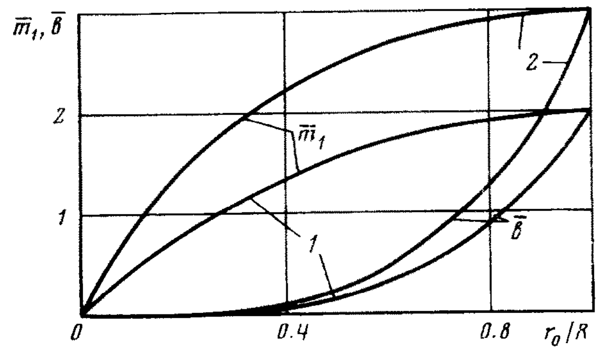

Fig. 2

3. **Equations of longitudinal oscillations of a structure with a hydrodynamic damper.** We denote the length of the structure by l, the running mass without fluid by $\mu(x)$ and the compressive (tensile) stiffness by $EF(x)$. We assume that the reference frame of the lower lid of the shell is situated in the plane $x = x_1$. The natural forms η_j and frequencies σ_j of the simultaneous longitudinal oscillations of the free construction regarded as a rod, and of the axisymmetric buckling oscillations of the shell with a fluid at $\zeta = 0$, can be found by solving the problem [3]

$$\frac{d}{dx}\left(EF\frac{d\eta}{dx}\right) + \sigma^2\left[\mu\eta + \sum_{n=1}^{\infty}\delta(x - x_1)\frac{m_{n0}\omega_{n0}^2}{\omega_{n0}^2 - \sigma^2}\eta\right] = 0, \quad m_{n0} = \frac{\lambda_{n0}^2}{\mu_{n0}}$$

$$\eta'(0) = \eta'(l) = 0$$

where m_{n0} are the masses of the oscillators equivalent to the shell containing a fluid and $\delta(x - x_1)$ is the Heaviside function. The conditions of orthogonality of the natural forms of oscillation have the form

$$\int_0^l \mu\eta_i\eta_j dx + m\eta_i(x_1)\eta_j(x_1) + \eta_i(x_1)\sum_{n=1}^{\infty} m_{n0}\overset{\circ}{g}_{nj} + \tag{3.1}$$

$$\eta_j(x_1)\sum_{n=1}^{\infty} m_{n0}\overset{\circ}{g}_{ni} + \sum_{n=1}^{\infty} m_{n0}\overset{\circ}{g}_{ni}\overset{\circ}{g}_{nj} = 0 \quad (i \neq j)$$

$$\overset{\circ}{g}_{nj} = g_{nj} - \eta_j(x_1), \quad g_{nj} = \frac{\omega_{n0}^2}{\omega_{n0}^2 - \sigma_j^2}\eta_j(x_1) \tag{3.2}$$

Let us expand the longitudinal displacements of the body of the structure into a series

$$u(x, t) = \sum_{j=1}^{\infty} q_j(t)\eta_j$$

and use the coefficients q_j as generalized coordinates of the system structure-hydrodynamic damper. The fluid displacements potential in the shell with a damper can be written in the form

$$\chi = \sum_{j=1}^{\infty} q_j\eta_j(x_1)x + \sum_{n=1}^{\infty} u_n\Phi_n + \zeta V \tag{3.3}$$

When the motion of the reference frame of the shell is given, the equations (2.4), with $\zeta = 0$ and (3.2) taken into account, give

$$u_n = \sum_{j=1}^{\infty} \frac{\lambda_{n0}}{\mu_{n0}}\overset{\circ}{g}_{nj}q_j \tag{3.4}$$

Substituting (3.4) into (3.3) we obtain

$$\chi = \sum_{j=1}^{\infty} q_j\left[\eta_j(x_1)x + \sum_{n=1}^{\infty}\frac{\lambda_{n0}}{\mu_{n0}}\overset{\circ}{g}_{nj}\Phi_n\right] + \zeta V \tag{3.5}$$

Let us write the expression for the kinetic energy of the structure with a hydrodynamic damper

$$T = \frac{1}{2}\int_0^l \mu\left[\sum_{j=1}^{\infty} q_j^{\cdot}\eta_j\right]^2 dx + \frac{1}{2}\rho\int_\tau \left(\nabla\frac{\partial\chi}{\partial t}\right)^2 d\tau$$

Substituting into this expression the displacement potential (3.5) and using the conditions of orthogonality (3.1), we obtain

$$T = \frac{1}{2}\sum_{j=1}^{\infty} a_j q_j^{\cdot 2} + \zeta^{\cdot}\sum_{j=1}^{\infty}\varepsilon_j q_j^{\cdot} + \frac{1}{2}b\zeta^{\cdot 2} \tag{3.6}$$

$$a_j = \int_0^l \mu\eta_j^2 dx + \sum_{n=1}^{\infty} m_{n0}q_{nj}^2, \quad \varepsilon_j = \sum_{n=1}^{\infty}\frac{\lambda_{n0}\varepsilon_{n0}}{\mu_{n0}}g_{nj}$$

Using the orthogonality conditions (3.1) we can reduce the expression for the potential energy of the structure to the form

$$\Pi = \frac{1}{2}\sum_{j=1}^{\infty} a_j\sigma_j^2 q_j^2 + \frac{1}{2} b\omega_\zeta^2\zeta^2$$

and now we can write the Lagrange's equation

$$a_j(q_j^{\cdot\cdot} + \sigma_j^2 q_j) + \varepsilon_j\zeta^{\cdot\cdot} = Q_j \quad (j = 1, \ldots, \infty)$$

$$b(\zeta^{\cdot\cdot} + \omega_\zeta^2\zeta) + \sum_{j=1}^{\infty}\varepsilon_j q_j^{\cdot\cdot} = 0$$

in which the generalized forces Q_j can be found in the usual manner.

The form in which the equations of motion are given, is the most suitable one for investigating the influence of the hydrodynamic damper on the dynamic properties of the structure. Generalization to the case of a structure with any number of shells and hydrodynamic dampers presents no difficulties.

4. Analysis of the effectiveness of the hydrodynamic vibration damper. Considering the first of the problems formulated above we note, that the most important aim is to reduce the amplitude of the oscillations of the shell at the frequency of the first mode, since the main bulk of the oscillating fluid mass corresponds to this mode. The damper should therefore be tuned to this oscillation frequency.

Let us consider the energy dissipated in the damper. We neglect the scattering of the energy in the shell and fluid, and the influence of the higher order modes. From (2.4) we obtain the equations of motion of the shell when the reference frame moves according to the law $u(t) = u_0 e^{i\omega t}$

$$\begin{aligned} \mu_{10}(u_1^{\cdot\cdot} + \omega_{10}^2 u_1) + \varepsilon_{10}\zeta^{\cdot\cdot} &= -\lambda_{10}u^{\cdot\cdot} \\ b(\zeta^{\cdot\cdot} + g\omega\zeta^{\cdot} + \omega_\zeta^2\zeta) + \varepsilon_{10}u_{10}^{\cdot\cdot} &= -\varepsilon_0 u^{\cdot\cdot} \end{aligned} \qquad (4.1)$$

where g is the relative damping coefficient of the damper.

Using the properties of the invariant points of the resonance curves [2] we obtain, for the system (4.1), the following conditions of the best tuning of the damper in terms of the acceleration:

$$\frac{\omega_\zeta^2}{\omega_{10}^2} = 1 - \frac{\alpha}{2}, \quad \alpha = \frac{\varepsilon_0\varepsilon_{10}}{b\lambda_{10}}$$

If in addition we choose the optimal damping in the damper, then we obtain the following expression for the maximum amplitudes of the shell resonance curve in terms of the acceleration:

$$u_{1*}^{\cdot\cdot} = \frac{\lambda_{10}}{\mu_{10}}\frac{2-\alpha-\sqrt{(2-\alpha)\varkappa}}{\sqrt{(2-\alpha)\varkappa}-\varkappa}|u^{\cdot\cdot}|, \quad \varkappa = \frac{\varepsilon_{10}^2}{b\mu_{10}}$$

In the absence of hydrodynamic damper, such resonance amplitude of the accelerations is obtained when the relative damping coefficient of the oscillations of the shell with fluid, for the first mode, is

$$g_{10} = \frac{\sqrt{(2-\alpha)\varkappa} - \varkappa}{2 - \alpha - \sqrt{(2-\alpha)\varkappa}} \tag{4.2}$$

It can be shown that for the cylindrical shells with hollow bottoms we have $\alpha \approx \varkappa \approx m_1 / m$. Clearly, it is very easy to ensure that e.g. $\alpha \approx \varkappa = 0.25$, for which we obtain $g_{10} = 0.39$. We note, for the purposes of comparison, that the relative damping coefficients of the shells with fluid are, as a rule, much smaller than 0.01 [3]. It follows that use of the hydrodynamic damper with optimal parameters makes it possible to increase the damping of the shell relative to the first mode of oscillation by tens of times.

Let us consider the second problem. Let the damper be tuned to the frequency of the j-th tone of longitudinal oscillations of the structure. We write the corresponding equations of oscillations in the form

$$a_j (q_j^{\cdot\cdot} + \sigma_j^2 q_j) + \varepsilon_j \zeta^{\cdot\cdot} = Q_j, \quad b(\zeta^{\cdot\cdot} + g\omega\zeta^{\cdot} + \omega_\zeta^2\zeta) + \varepsilon_j q_j^{\cdot\cdot} = 0$$

The damping in the structure is neglected for simplicity.

The conditions of optimal tuning of the damper and the expression for the smallest resonance amplitudes of accelerations of the structure possible under the optimal damping, have the form

$$\frac{\omega_\zeta}{\sigma_j} = 1, \quad q_{j*}^{\cdot\cdot} = \frac{Q_j}{a_j} \frac{2 - \sqrt{2\varkappa_j}}{\sqrt{2\varkappa_j} - \varkappa_j}, \quad \varkappa_j = \frac{\varepsilon_j^2}{b a_j}$$

The effectiveness of the hydrodynamic damper is determined by the magnitude of the parameter $\varkappa_j$. For cylindrical shells with hollow bottoms we have [3]

$$m_{10} \approx m, \quad m_{n0} / m \ll 1 \quad (n = 2, \ldots, \infty) \tag{4.3}$$

If (4.3) holds, then

$$\frac{\lambda_{10}\varepsilon_{10}}{\mu_{10}} \approx \varepsilon_0, \quad \frac{\lambda_{n0}\varepsilon_{n0}}{\varepsilon_0 \mu_{n0}} \ll 1 \quad (n = 2, \ldots, \infty)$$

From this we conclude, taking into account (3.2) and (3.6), that the hydrodynamic damper should preferably be set up in a shell the partial frequency ω_{10} of the lower mode of which is nearly equal to the frequency σ_j of the longitudinal oscillations of the structure. In this case we have

$$\varkappa_j = \frac{\varepsilon_0^2}{b a_j} g_{1j}^2 = \frac{m_1}{a_j} \frac{\omega_{10}^4 \eta_j^2(x_1)}{(\omega_{10}^2 - \sigma_j^2)^2} \tag{4.4}$$

We see from (4.4) that $\varkappa_j$ can assume relatively large values. For example, for a structure consisting of six shells filled with fluid we can ensure that $m_1 \eta_j^2(x_1) / a_j = 0.02$ and even when $\omega_{10}^2 = 2\sigma_j^2$, , we have $\varkappa_j = 0.08$.

Similarly to (4.2), we can introduce a relative damping coefficient of the structure with respect to the j-th mode, equivalent to the effect of the damper

$$g_j = \frac{\sqrt{2\varkappa_j} - \varkappa_j}{2 - \sqrt{2\varkappa_j}}$$

When $\varkappa_j = 0.08$, we have $g_j = 0.2$. For thin-walled structures containing large masses of fluid, the values of the relative damping coefficients fall, as a rule, within the range 0.01 – 0.05 [3]. The hydrodynamic damper may be used to remove a dynamic instability of the longitudinal oscillations of such structures.

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